Full-Depth Global Estimates of Ocean Mesoscale Eddy Mixing from Observations and Theory

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Key Points:

- A new parameterisation for mesoscale eddy mixing that combines mixing-length and flow-suppression theory with vertical modes.
- Global, full-depth mesoscale eddy mixing estimates are obtained from observations of temperature, salinity, pressure and eddy kinetic energy.
- Mesoscale eddy mixing is surface intensified and strongly influenced by the vertical stratification of ocean density.

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Abstract

Mixing by mesoscale eddies profoundly impacts climate and ecosystems by redistributing and storing dissolved tracers such as heat and carbon. Eddy mixing is parameterized in most numerical models of the ocean and climate. To reduce known sensitivity to such parameterizations, observational estimates of mixing are needed. However, logistical and technological limitations obstruct our ability to measure global time-varying mixing rates. Here, we extend mixing length theory with mean-flow suppression theory, and first surface modes, to estimate mixing from readily-available observational-based climatological data, of salinity, temperature, pressure and eddy kinetic energy at the sea surface. The resulting full-depth global maps of eddy mixing can reproduce the few available direct estimates and confirm the importance of mean-flow suppression of mixing. The results also emphasize the significant effect of eddy surface intensification and its relation to the vertical density stratification. These new insights in mixing dynamics will improve future mesoscale eddy mixing parameterizations.

Plain Language Summary

Large whirls of hundreds of kilometers can mix water with different temperatures, salinity and other properties. These whirls are called “mesoscale eddies” and are very difficult to include in numerical simulation of ocean and climate. Therefore, we include them using a simplified representation: a parameterization. These parameterizations need as input, the strength with which these eddies mix. Ideally, we would thus measure these mixing strengths globally and over the full depth of the ocean. However, this is impossible due to technological, logistical and financial limitations. To avoid these limitations, we instead indirectly estimate mixing from variables that we can measure globally over the full depth of the ocean. We here present a new way to indirectly estimate mixing from widely available observations of temperature, salinity, pressure and surface eddy kinetic energy. This results in 3-dimensional maps of eddy mixing strengths. We find that eddies mix much stronger near the surface than in the deep ocean, and that this is partly caused by the vertical stratification of ocean density. These new insights and maps can be used to improve mixing parameterizations and thus significantly improve all kinds of calculations that are important for the Earth’s climate and ecosystems.

1 Introduction

Instability of the large-scale density field produces geostrophically-balanced, mesoscale ocean eddies (Gill et al., 1974; Wunsch & Ferrari, 2004), which are central for ocean mass and tracer transport (Gnanadesikan et al., 2015; Busecke & Abernathey, 2019; Jones & Abernathey, 2019; Busecke et al., 2014) The eddies vary in size from kilometers to hundreds of kilometers and as such are unresolved in many global ocean and climate models (Chelton et al., 1998). The eddies are accordingly parameterized in the models (Meijers, 2014; Fox-Kemper et al., 2019; Jansen et al., 2019; Hallberg, 2013). But the simulations are often sensitive to the choice of parameterizations (Jones & Abernathey, 2019; Ferreira et al., 2005; Sijp et al., 2006; Pradal & Gnanadesikan, 2014).

Having proper three dimensional observations of eddy mixing would greatly aid in the choice of parameterizations, reducing model uncertainty. Direct observations have been made in tracer release experiments (Ledwell et al., 1993, 1998), and also estimated using Lagrangian drifters and subsurface floats (Zhurbas & Oh, 2004; LaCasce et al., 2014; Roach et al., 2018), satellite data (Holloway, 1986; R. P. Abernathey & Marshall, 2013; Klocker & Abernathey, 2013; Busecke & Abernathey, 2019), hydrographic data (Chapman & Sallée, 2017), inverse methods (Zika et al., 2010; Groeskamp et al., 2017; Hautala, 2018) and salinity anomalies (Cole et al., 2015). These studies have significantly increased our understanding of eddy size, eddy kinetic energy (EKE) and mixing suppression by mean flows. Broadly speaking, the studies indicate enhanced mixing near western boundary currents and the

Antarctic Circumpolar Current, and reduced mixing in the eastern subtropical-gyres and at high latitudes. However, almost none of these observational-based studies offer global and/or full-depth coverage; rather they are local, confined to the surface, or limited to the area significantly sampled by Argo floats. The one global study that we know of provides only estimates integrated over the mixed layer and the abyss (Groeskamp et al., 2017).

Here we propose a method to estimate eddy mixing using satellite observations and readily available observations of Absolute Salinity S_A, Conservative Temperature Θ, pressure p and the surface EKE. This involves projecting the surface velocities to depth using recently-derived vertical structure functions. The resulting global, full-depth diffusivity maps agree well with previous observations and indirect estimates. The vertical density stratification (N^2) dictates the extent of surface intensification of eddy mixing and, in turn, of mixing at depth. These new insights will aid the next steps in understanding, constructing and constraining eddy mixing parameterizations.

2 The eddy diffusivity parameterizations

We here study the mixing by along-isopycnal diffusion of passive tracers by mesoscale eddies, which we represent with a turbulent isopycnal mesoscale eddy diffusivity K. The foundation is an estimate based on mixing length theory (Prandtl, 1925):

$$K_\mathrm{MLT} = \Gamma \; u_\mathrm{rms} \; L_\mathrm{mix} \tag{1}$$

Here Γ is the mixing efficiency, u_rms the rms geostrophic velocity and L_mix the mixing length scale. Such a relation has been used for both atmospheric (Bretherton, 1966) and oceanic modelling (Holloway, 1986; Marshall & Adcroft, 2010; R. P. Abernathey & Marshall, 2013; Klocker & Abernathey, 2013; Naveira Garabato et al., 2015). Following the realization that mixing is suppressed in the presence of a mean flow, the above estimate was modified. For the case of a zonal mean flow, this is (Ferrari & Nikurashin, 2010; Klocker et al., 2012):

$$K \quad = \quad \frac{K_\mathrm{MLT}}{1 + k^2\gamma^{-2}\left(c_w - U\right)^2} \tag{2}$$

Here γ is the reciprocal of an eddy decorrelation time (s^{-1}), k the zonal (eddy) wavenumber, c_w an eddy drift speed and U the mean velocity.

2.1 Depth Dependent K_MLT

Eq. (2) has been applied previously at the sea surface (Klocker & Abernathey, 2013), but here we extend it to depth by using the depth-dependent velocity $U = U(z)$ in the denominator (Ferrari & Nikurashin, 2010; Klocker et al., 2012). The velocity can be obtained from integrating the thermal wind relation downward from the sea surface. K_MLT also depends on depth, through its dependence on u_rms. Assuming the vertical structure is separable, the components of the eddy velocity can be expressed as $(u', v') = \phi(z)\;(u'_0, v'_0)$, where $\phi(z)$ is a vertical structure function and the zero subscript velocities are the surface values. The $u_\mathrm{rms}(z)$ is then given by:

$$u_\mathrm{rms} \quad = \quad \sqrt{(u'^2 + v'^2)} = \phi(z)\sqrt{2\mathrm{EKE}_0}, \tag{3}$$

which can be obtained from the surface EKE_0.

The function $\phi(z)$ in turn can be found by solving a vertical structure equation which has the vertical stratification N^2 as the only input (Charney, 1971; Gill, 1982; Wunsch, 2015). The resulting eigenmodes (the "baroclinic modes") were traditionally found assuming a flat bottom boundary. The first baroclinic mode is familiar in oceanography, having opposed flow at the surface and bottom. However, observations from current meters suggest that bathymetry affects time-dependent motion throughout the water column, in most regions of the ocean (de La Lama et al., 2016). This can be taken into account by imposing a no horizontal flow condition at the bottom, i.e. $\phi(z = -H) = 0$ (LaCasce, 2017). The gravest resulting ϕ (the "first surface mode") closely resembles the gravest empirical orthogonal function from the current meter observations (de La Lama et al., 2016; LaCasce & Groeskamp, 2020). The latter accounts for more than 50% of the variance in many locations. Thus we use the first surface mode to represent the dominant vertical structure of the horizontal geostrophic eddies (Appendix B).

Bottom-trapped (topographic wave) modes (Rhines, 1970; Thompson & Luyten, 1976) may also contribute significantly to subsurface mixing. Such modes have a maximum near the bottom that decays with height, and the associated mixing would be similarly bottom-intensified. However, it is presently unknown how to estimate the topographic wave field from surface data (or if that is even possible). This is should be the subject of future work.

We must also specify the mixing length, L_{mix} (Eq. 1). A consistent choice is the first Rossby radius of deformation, L_{d}, the eigenvalue associated with the first baroclinic mode. Ocean eddies propagate westward, in most regions outside of the Southern Ocean (where, due to advection by the Antarctic Circumpolar Current, they drift eastward instead) (Chelton & Schlax, 1996; Chelton et al., 1998). The propagation speed is consistent with that of long Rossby waves, if one uses the deformation radius associated with the first surface mode (LaCasce & Groeskamp, 2020). As such, the surface mode radius is a reasonable choice for eddy scale.

The radius, which is inversely proportional to the Coriolis parameter, becomes infinite at the equator. Thus one uses an alternate estimate, based on the "equatorial beta plane", at low latitudes (Chelton et al., 1998). Specifically, we use the expression of Hallberg (2013):

$$L_{\text{d}} = \frac{c_1}{\sqrt{f^2 + 2c_1\beta}}. \tag{4}$$

where c_1 is the first surface mode gravity wave phase speed, f is the Coriolis parameter and $\beta = df/dy$ is its latitudinal gradient.

2.2 The Suppression Factor

Estimates of c_w (Eq. 2) are often made using Hovmöller diagrams of the sea surface height. Instead, we exploit the fact that over most of the ocean eddy drift speeds are well-approximated by the surface mode phase speed, Doppler shifted by the time- and depth-averaged velocity (Klocker & Abernathey, 2013; Klocker & Marshall, 2014; Chapman & Sallée, 2017). This yields $\mathbf{c_w}(x, y) = \left(\overline{U}^{z,t} - \beta L_{\text{d}}^2,\ \overline{V}^{z,t}\right)$. Again, the velocities U and V are obtained from thermal wind, and L_{d} is the first surface radius. An alternate expression can be obtained assuming a two layer ocean(Wang et al., 2016), but the present version is more appropriate with continuous stratification. Finally, we write the wavenumber as $k = 2\pi/L_{\text{d}}$. Taken together, this yields:

$$K = \underbrace{\Gamma\ \phi(z)\ \sqrt{2\text{EKE}_0}\ L_{\text{d}}}_{K_{\text{MLT}}} \times \min\left(S^x, S^y\right), \tag{5}$$

with

$$S_x = \frac{1}{1 + \frac{4\pi^2}{\gamma^2 L_{\text{d}}^2}\left(\delta v\right)^2}, \quad S_y = \frac{1}{1 + \frac{4\pi^2}{\gamma^2 L_{\text{d}}^2}\left(\delta u - \beta L_{\text{d}}^2\right)^2}, \tag{6}$$

where $\delta v = \overline{v}^{z,t} - v(z)$ and $\delta u = \overline{u}^{z,t} - u(z)$. Note that U_0 and V_0 have cancelled out, avoiding the introduction of errors through an estimate of the surface velocity. As such, δu and δv are entirely determined from hydrography, $(S_{\text{A}}, \Theta, p)$.

Suppression theory provides a means for estimating cross-stream diffusivities, as along-stream transport is dominated by advection (Ferrari & Nikurashin, 2010). Following Klocker and Abernathey (2013); Chapman and Sallée (2017), we use the minimum suppression factor, i.e. $\min(S^x, S^y)$, rather than the value perpendicular to the large-scale mean flow (Zhurbas & Oh, 2004). The latter was attempted, but yielded noisier results due to uncertainties in the rotation angle. Finally, we note that γ is used as a fitting parameter, as discussed later.

3 Collecting the ingredients

Thus we require only observations of S_A, Θ, p and EKE_0 to calculate K. The former were obtained from annual mean fields from the World Ocean Atlas 2018 (Garcia et al., 2019), gridded climatology, while the EKE_0 comes from CMEMS (Copernicus Marine Environment Monitoring Service) operational delayed-time sea surface geostrophic velocity anomalies derived from satellite altimetry (Pujol et al., 2016; Taburet et al., 2019) (Appendix A). The mixing parameter was set to be $\Gamma = 0.35$ (Klocker & Abernathey, 2013).

The decay rate, γ, was obtained by fitting K to the diffusivity estimates obtained from the North Atlantic Tracer Release Experiment (NATRE) and Diapycnal and Isopycnal Mixing Experiment in the Southern Ocean (DIMES). The parameter was found via a least squares fit using both sets of data (using only estimates indicated with black colors in Fig. 1). We find $\gamma^{-1} = 1.68$ days, which is comparable to, but also somewhat smaller than the 4 days found previously for the surface (Klocker & Abernathey, 2013).

4 Comparing to observations

The resulting diffusivity profiles compare well to those obtained in NATRE and DIMES (Fig. 1). The present estimate captures the vertical decay in the NATRE profile, and also the subsurface maximum observed in the DIMES experiment. Notably, in both cases including the mean flow suppression factor significantly improves the results.

Other estimates are also shown for comparison. The semi-global estimates of Cole et al. (2015) somewhat overestimate the diffusivities, while the global inverse estimates of Groeskamp et al. (2017) underestimate them. An additional curve is included in each case showing the vertical structure if the traditional (flat-bottom) baroclinic mode is used instead of the surface mode (Appendix B). This yields a mid-depth minimum and thus a very different vertical structure than observed.

4.1 Global maps

The factor indicating mean flow suppression of the mixing, given in (6), is mapped in Fig. (2). Suppression is enhanced in the cores of the western boundary currents and reduced on their flanks. It is pronounced in the Antarctic Circumpolar Current, over large swaths of the Southern Ocean. Suppression is also strongly evident at low latitudes, reflecting the large deformation radius there. Suppression is correspondingly weaker in the high latitudes, where L_d is small.

The suppression factor helps understand the spatial variability exhibited by the diffusivity, shown in Fig. 3. For example, K is weaker in the core of the Gulf Stream but enhanced to its south, while diffusivities in the equatorial region are smaller than in the subtropical gyres. Diffusivities are large in the Agulhas system, in the South Equatorial Current in the Indian Ocean and in the western Pacific. Yet, diffusivities are smaller at high latitudes as the smaller L_d yields a shorter mixing length. Previously-noted features such as the mixing desert in the North Pacific and Subtropical Southern Atlantic (Klocker & Abernathey, 2013; R. P. Abernathey & Marshall, 2013), are also clearly seen. Meanwhile the estimates in the NATRE and DIMES regions, indicated by the squares in the eastern North Atlantic and the South Pacific, respectively, compare well to those described previously (Klocker & Abernathey, 2013; R. P. Abernathey & Marshall, 2013; Chapman & Sallée, 2017) (Fig. 1).

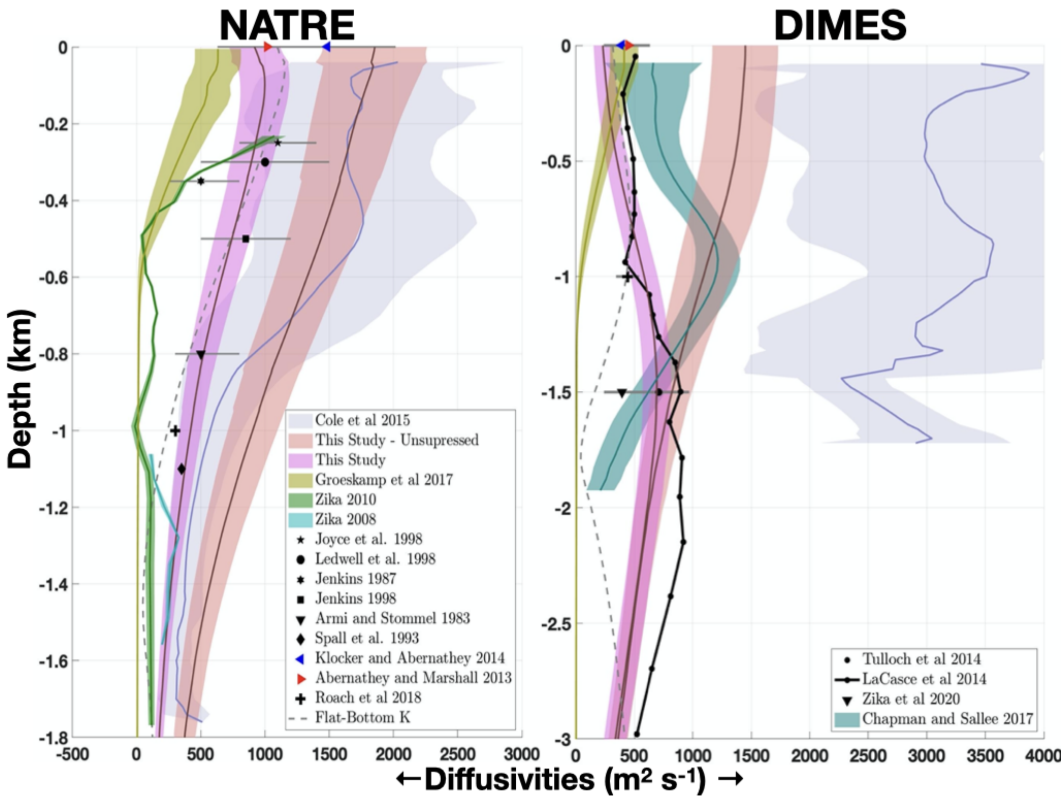

Figure 1. Comparing the present results against direct observations (black points) (Joyce et al., 1998; Ledwell et al., 1998; Jenkins, 1987, 1998; Armi & Stommel, 1983; Spall et al., 1993; Tulloch et al., 2014; Zika et al., 2020) and indirect estimates (Cole et al., 2015; Zika et al., 2010; Zika & McDougall, 2008; Klocker & Abernathey, 2013; R. P. Abernathey & Marshall, 2013; LaCasce et al., 2014; Groeskamp et al., 2017; Roach et al., 2018; Chapman & Sallée, 2017) in the NATRE (left) and DIMES (right) region.

Mean flow suppression also varies with depth (Fig. 2b), affecting the vertical structure of the diffusivities (Fig. 3b). Strong suppression is observed at all depths in the tropics, but suppression is intensified near the surface and weaker at depth at mid-latitudes. It is for this reason the diffusivity exhibits a subsurface maximum in for example the Southern Ocean (Fig. 3b), including the DIMES region (Fig. 1) (R. Abernathey et al., 2010; Klocker et al., 2012; LaCasce et al., 2014; Tulloch et al., 2014).

The vertical variation of the diffusivity also depends on the vertical structure of the eddy kinetic energy, represented by the function $\phi(z)$. As noted, this is obtained by applying theoretical arguments to an observationally based gridded hydrographic climatology (LaCasce & Groeskamp, 2020). We illustrate this by mapping the depth at which ϕ has decreased by an e-folding scale from its surface value ($\phi(z) = 1/e = 0.37$, Fig. 4a), and show representative vertical profiles of $\phi(z)$ (Fig. 4b). The e-folding depth is small in shallow regions, for instance along the continental slopes. It is large where the stratification is weak, notably near Antarctica and in the Labrador Sea, and smaller where the stratification is strong, as in the subtropical gyres (10-30°N and 10-30°S). That the diffusivity is strongly surface intensified is also clear in the NATRE region (Fig. 1).

There is significant longitudinal variation however, for example in the South Atlantic. The larger e-folding depth in the west stems from the intrusion of Antarctic Bottom Water.

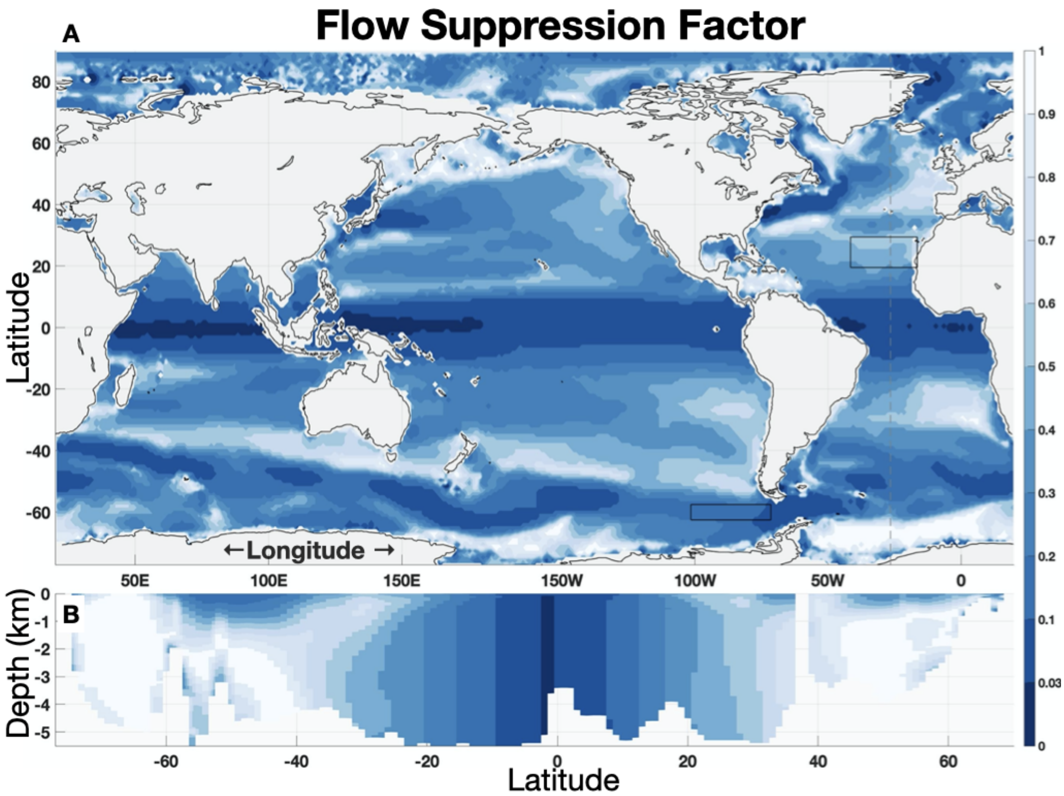

Figure 2. The suppression factor at the surface (Eq. 6) (A) and for a north-south transect in the Atlantic ocean (B), as indicated by the meridional grey dashed line in (A). NATRE and DIMES are indicated by squares.

This increases the deep ocean stratification, causing a more gradual vertical decay of ϕ (to satisfy the bottom boundary condition) than in the east side of the basin. This contrast is consistent with data from two WOCE current meters, at the locations indicated by the star and diamond in the map. The first Empirical Orthogonal Functions (EOF), derived from the horizontal velocities (Fig. 4b), are plotted in the insert. The eastern current meter (though just to the east of the shallow portion of the map) has an EOF which decays more rapidly with depth that the western current meter (the starred profile). This leads to stronger surface intensification in the east and thus larger mixing rates at depth in the western South Atlantic than in the east.

5 Summary

A new parametrization for the along-isopycnal mesoscale eddy diffusivity is presented. This novelly extends the Prandtl (1925) mixing length theory employing mean-flow suppression theory (Ferrari & Nikurashin, 2010) and the theory of vertical modes over bathymetry (LaCasce & Groeskamp, 2020). The parameterisation is applied to widely- and readily-available observations of S_{A}, Θ, p and surface EKE to produce a global, full-depth map of along-isopycnal mesoscale eddy diffusivity. The estimated diffusivities exhibit strong spatial variations and are in line with previous surface estimates, and also agree well with subsurface profiles obtained from experiments in the eastern North Atlantic and the Southern Ocean.

The adopted surface mode vertical structure, while supported by observations, has not yet been widely adopted. An exception is the Geophysical Fluid Dynamics Laboratory

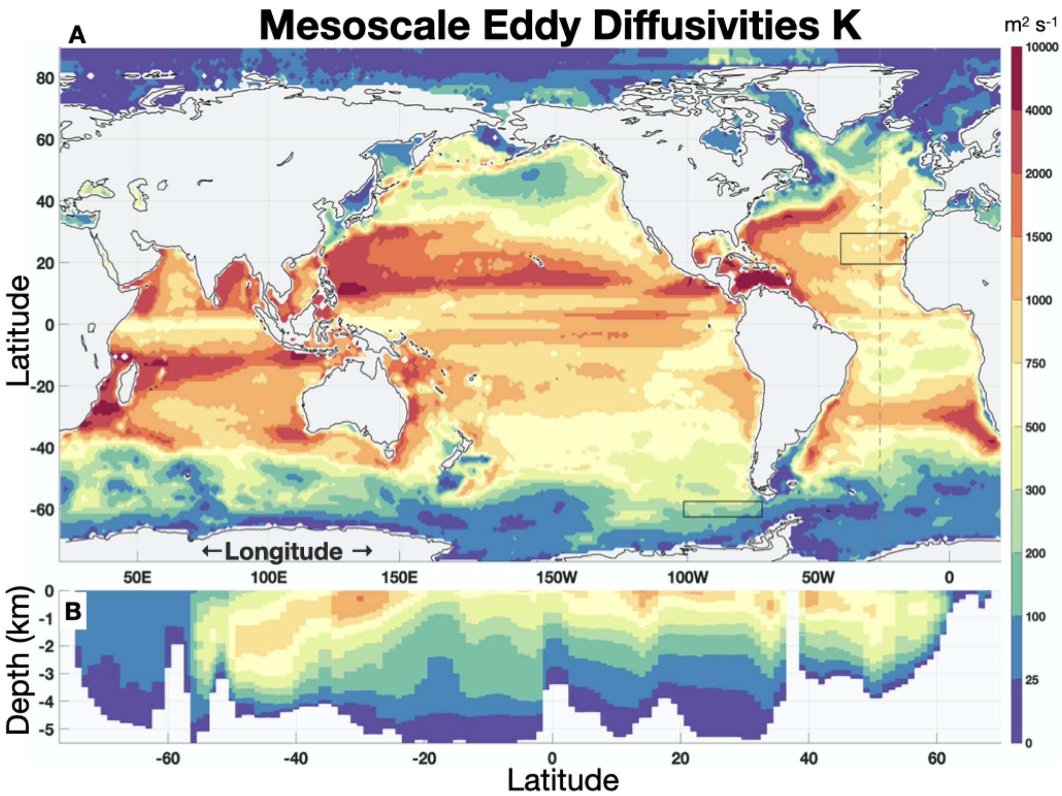

Figure 3. The diffusivity K at the ocean surface (A) and for a north-south transect in the Atlantic ocean (B) along the grey dashed line in (A). The NATRE and DIMES regions are indicated by rectangles.

(GFDL) OM4.0 numerical model, which employs a first baroclinic mode with no slip imposed at the bottom when representing the vertical structure of the diffusivity (Adcroft et al., 2019). With strong surface stratification, relative to the abyssal stratification, the surface mode is strongly surface-intensified. This is in line with previous studies which also found indications of surface intensification (Groeskamp et al., 2017; Canuto et al., 2019). Consequently, large scale thermohaline and wind forcing that alters surface stratification determines how mixing varies with depth. Thus while eddies may alter the stratification (Dewar, 1986), stratification also impacts eddy mixing.

The diffusivities derived here represent mesoscale mixing of tracer (Redi, 1982), yet the same eddies also mix mass between pairs of density surfaces. This is called the temporal residual-mean velocity in height-coordinate models and bolus-velocity in density coordinate models (T. J. McDougall & McIntosh, 2001). The tracer and "mass" diffusivities are known to differ, but are related through theoretical considerations as described by Smith and Marshall (2009). Applying their theory to the presented diffusivities may provide a way forward to use the results of this study for both temporal residual-mean and bolus-velocity transports.

The development of mixing parameterizations that are able to respond to changing state of the ocean remains a challenge for numerical modeling (Fox-Kemper et al., 2019). The present parameterisation, based on the ocean state, provides a way forward to overcome this challenge. This will in turn much improve numerical modeling of ocean physics, biogeochemistry and future climate.

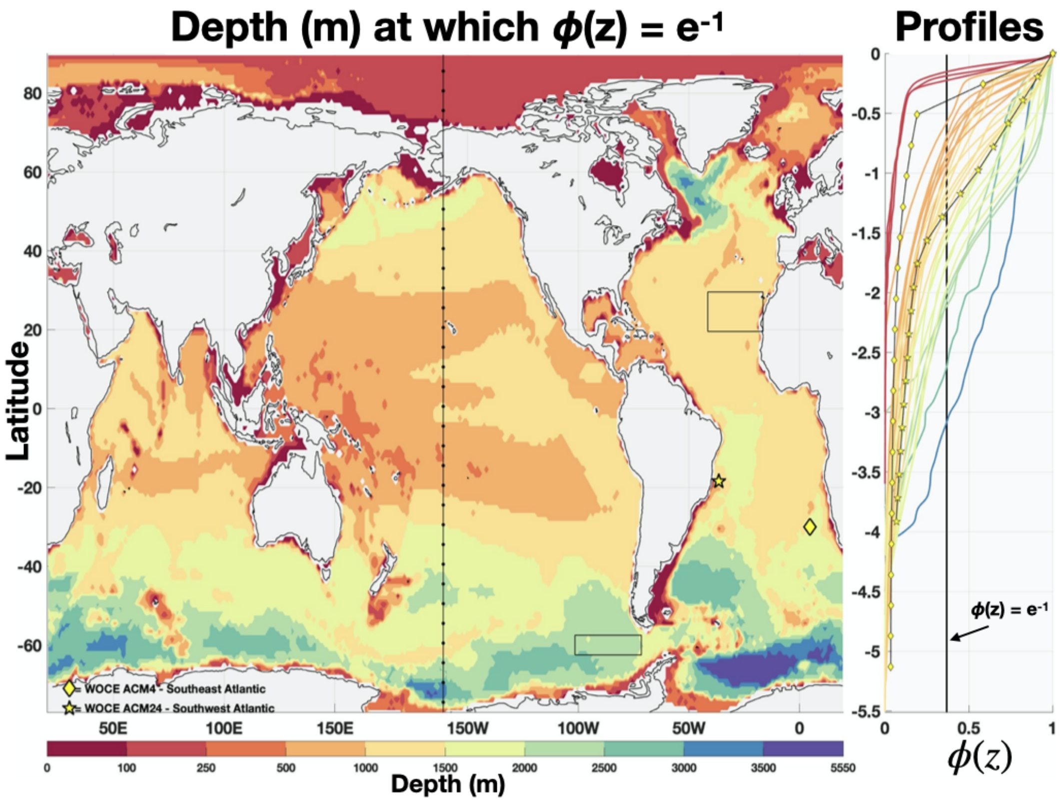

Figure 4. The e-folding depth for the geostrophic eddy velocity, i.e the depth at which $\phi(z) = 0.37$ (left). The star and diamond indicate the locations of the ACM24 (Durrieu De Madron & Weatherly, 1994) and ACM04 (Garzoli et al., 1996) current meter moorings, respectively. Examples of individual profiles of $\phi(z)$ (right). The profiles are from 160W, indicated by the black dotted line in the left panel, and the two current meters. The colors are the same as for the left panel.

6 Appendix A - the data

World Ocean Atlas *in situ* temperature and practical salinity are used to calculate Conservative Temperature Θ and Absolute Salinity S_{A} (T. J. McDougall, 2003; Graham & McDougall, 2013; T. J. McDougall et al., 2012; IOC et al., 2010) using the GSW software toolbox (T. McDougall & Barker, 2011), and are then interpolated to a 10 m vertical grid resolution using interpolation software of Barker and McDougall (2020). The resulting data is made statically stable using Barker and McDougall (2017), with a minimum stability given by Jackett and McDougall (1997). The resulting buoyancy frequency N^2 is smoothed with a 5-point running mean to filter out small scale oscillations.

The CMEMS multiple-satellite-merged data are daily, spanning from 1993 to present, and gridded at a spatial resolution of 0.25° in both zonal and meridional directions. The geostrophic currents are calculated using the geostrophic relations for latitudes outside the $\pm 5°$N band, and using a β-plane approximation of the geostrophic equations in the equatorial band (Lagerloef et al., 1999). The u_{rms} is defined here as the root mean square of the mean EKE, computed from the altimetric geostrophic velocity anomalies over the period 1993-01-01 – 2017-05-15, and is re-gridded onto the WOA grid before computing u_{rms} (Fig. 5b).

Current meter data is used in the East (mooring 4 of ACM04 (Garzoli et al., 1996) and West (mooring 4 of ACM24 (Durrieu De Madron & Weatherly, 1994)) Atlantic. Both moorings measure velocity at 4 depths ranging 900m to 3915m for ACM24, and from 210m

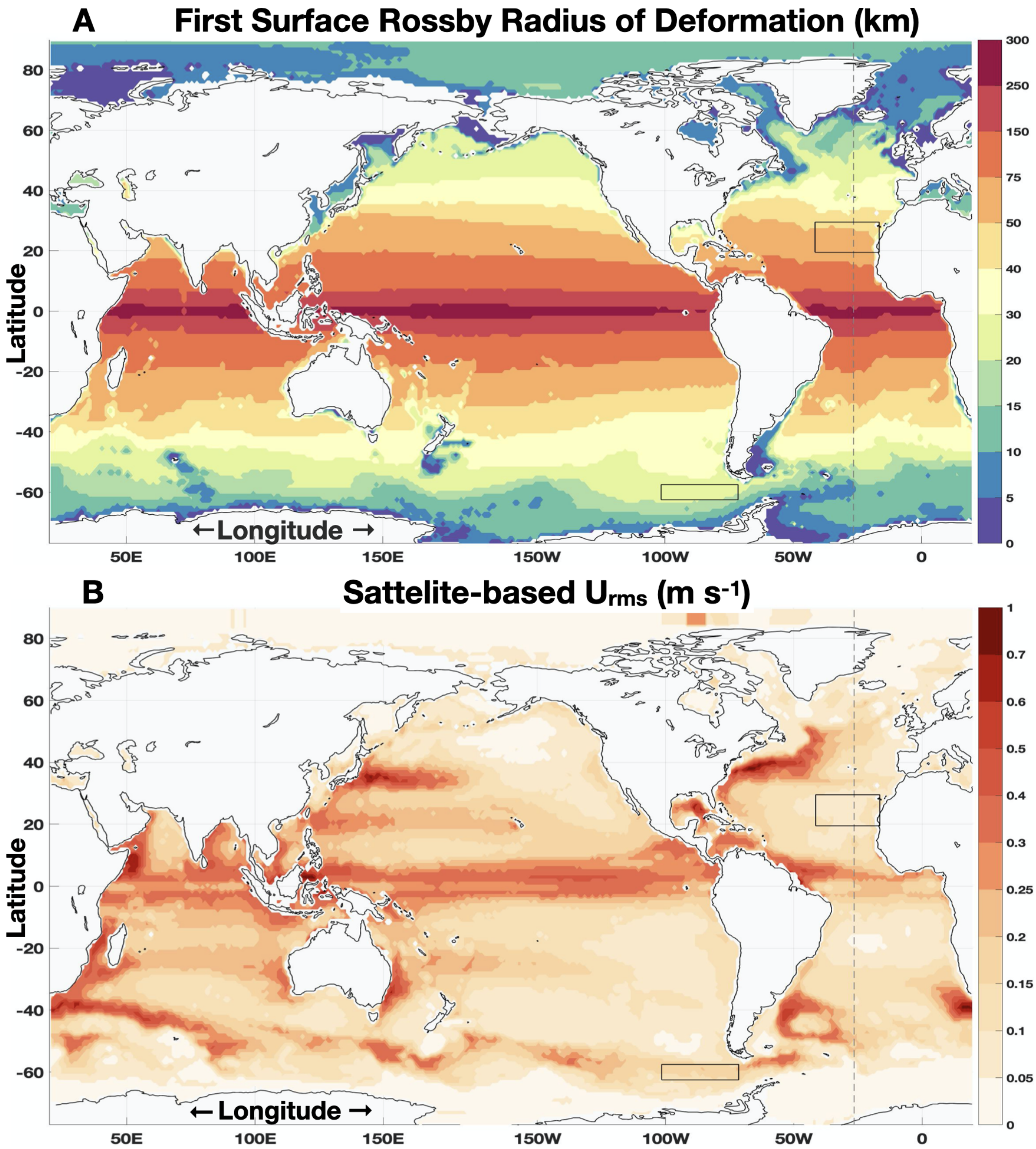

Figure 5. The first surface mode deformation radius L_{d} (A), and the annual mean root mean square (rms) geostrophic eddy velocity u_{rms} derived from sea surface height satellite data (B)

to 4092m for ACM04. The first EOF of the measured EKE explains 84% and 90% of the variance for AMC24 and AMC04, respectively. The modes are linearly interpolated to the surface, and normalised with the surface value. The results are interpreted as an indication that we find similar behaviour from observations of ocean currents as from rough-bottom modes derived using Surface Modes.

7 Appendix B - Solving for Φ and L_{d}.

The linear Quasi Geostrophic Potential Vorticity equation governs flows with small Rossby numbers. Assuming a plane wave solutions of the form $\sim \phi(z)\ \tilde{\Psi}e^{(ikx+iyl-i\omega t)}$ yields a differential equation for the vertical structure the horizontal flow $\phi(z)$ (Gill, 1982;

Pedlosky, 1987; Wunsch, 2015):

$$\frac{d}{dz}\left(\frac{f_0^2}{N^2}\frac{d\phi}{dz}\right) + \frac{1}{c^2}\phi = 0, \quad \text{with} \quad N^2(z) = g\left(\alpha\frac{\partial\Theta}{\partial z} - \beta\frac{\partial S_{\mathrm{A}}}{\partial z}\right) \tag{7}$$

Here $N(z)$ is the buoyancy frequency and f_0 is the mean Coriolis parameter. Solving the equation requires only climatological $(S_{\mathrm{A}}, \Theta, p)$ and boundary conditions. Traditionally Eq. (7) was solved assuming a rigid lid and a flat bottom, such that the vertical velocity $(\partial\phi/\partial z)$ vanishes at the upper and lower boundary $(z = 0, -H)$ (Kundu et al., 1975; Gill, 1982; Philander, 1978; Wunsch & Stammer, 1997; Nurser & Bacon, 2014). However, recent studies argue that bottom topography suppresses the deep flow (de La Lama et al., 2016; LaCasce, 2017), so that it is preferable to solve Eq. (7) with no horizontal flow at the bottom instead (i.e. $\phi(z = -H) = 0$). With realistic stratification, Eq. (7) is solved numerically using a fourth-order Runge-Kutta step to integrate downward from the surface from an initial guess, with adjustments to the eigenvalue made by using Newton's method until the bottom boundary condition is satisfied. The gravest resulting mode, the "First Surface Mode", resembles the "equivalent-barotropic" structure (Killworth, 1992) in that it decays from the surface to the bottom without changing sign. The surface mode also closely resembles the primary EOF from current meter observations, which often accounts for 50-90% of the variance (de La Lama et al., 2016; LaCasce & Groeskamp, 2020). The deformation radius is then given by Eq. (4) (Fig. 5a). We also solve Eq. (7) for the traditional flat-bottom boundary condition to obtain ϕ_{flat}. We use $|\phi_{\mathrm{flat}}|$, its associated deformation radius and a new fit of $\gamma^{-1} = 1.38$ days to obtain the flat-bottom estimate of K shown in Fig. 1.

Acknowledgments

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